

微分積分学第一 (LAS.M101-06)

多変数関数と偏微分

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高階の導関数 (1 変数)

▶ $I \subset \mathbb{R}$: 開区間 ; $f : I \rightarrow \mathbb{R}$

$$f''(x) = \frac{d^2 f}{dx^2} = \frac{d}{dx} \frac{df}{dx},$$

$$f'''(x) = \frac{d^3 f}{dx^3} = \frac{d}{dx} \frac{d^2 f}{dx^2},$$

⋮

$$f^{(m)}(x) = (f^{(m-1)})'(x) = \frac{d}{dx} \frac{d^{m-1} f}{dx^{m-1}}, \dots$$

2次導関数
2階微分

2次偏導関数

▶ $D \subset \mathbb{R}^2$: 領域 ; $f: D \rightarrow \mathbb{R}$

(4)

$$f_{xx} = (f_x)_x = \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \frac{\partial f}{\partial x}$$

$$f_{xy} = (f_x)_y = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \frac{\partial f}{\partial x}$$

$$f_{yx} = (f_y)_x = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \frac{\partial f}{\partial y}$$

$$f_{yy} = (f_y)_y = \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \frac{\partial f}{\partial y}$$

xy a 並に順序は同じ。

実は $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$

高階偏導関数

(3R)

(8=)

$$\frac{\partial^3 f}{\partial x^3} \quad \frac{\partial^3 f}{\partial x \partial y^2} \quad \frac{\partial^3 f}{\partial x \partial y \partial x} \quad \dots$$

順序交換は出来る

(4=)

$$\frac{\partial^2 f}{\partial x^3} \quad \frac{\partial^2 f}{\partial x^2 \partial y} \quad \frac{\partial^2 f}{\partial x \partial y^2} \quad \frac{\partial^2 f}{\partial y^2}$$

例

$$f(x, y) = x^3 - 3x^2y + y^5$$

$(\frac{\partial}{\partial x}, \frac{\partial}{\partial y})$

$$f_x = 3x^2 - 6xy$$
$$f_y = -3x^2 + 5y^4$$
$$f_{xx} = 6x - 6y$$
$$f_{yx} = -6x$$
$$f_{xy} = -6x$$
$$f_{yy} = 20y^3$$

この例 2" は $f_{xy} = f_{yx}$

偏微分の順序交換定理

f : 2変数関数

たいてい

$$f_{xy} = f_{yx}$$

・ 次回授業をのろふ

・ 正しい公式でか12..3番のはOK

例

$$f(x, y) = \sqrt{x^2 + y^2}$$

$$f_x = \frac{x}{\sqrt{x^2 + y^2}}$$

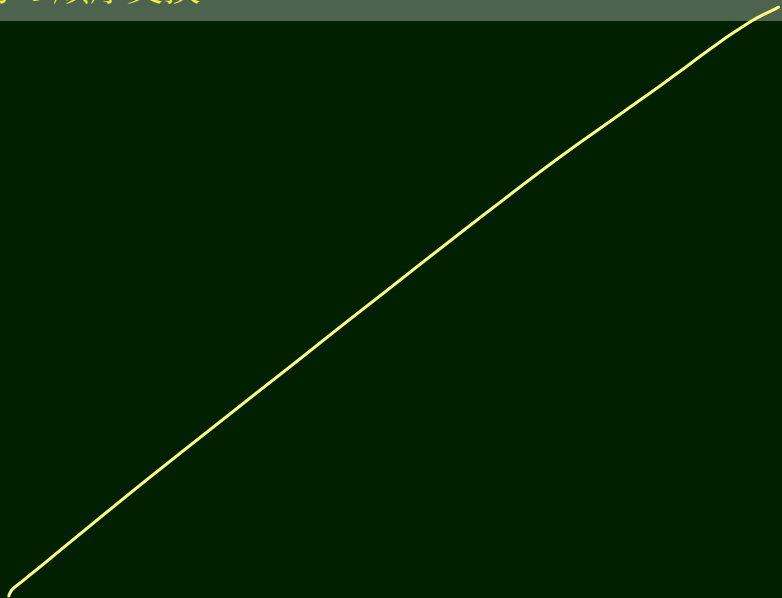
$$f_y = \frac{y}{\sqrt{x^2 + y^2}}$$

$$f_{xy} = \frac{-xy}{\sqrt{x^2 + y^2}^3}$$

$$f_{yx} = \frac{-yx}{\sqrt{x^2 + y^2}^3}$$

$$\left(\begin{array}{c} \sqrt{} \\ -2x \sqrt{} \end{array} \right)$$

偏微分の順序交換



例

$$f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

3変数関数

f_x (y, zは定数と見之) f_y

f_z

$-\frac{1}{r^2} \frac{\partial r}{\partial x}$

etc

(Laplaceの方程式)

② $f_{xx} + f_{yy} + f_{zz} = 0$

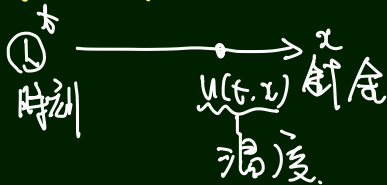
Δf Laplacian

例

$$u(t, x) = \frac{1}{2\sqrt{\pi ct}} \exp\left(-\frac{x^2}{4ct}\right) \quad (c \text{ は正の定数})$$

熱方程式 (1次元)

$$\frac{\partial u}{\partial t} = c \frac{\partial^2 u}{\partial x^2}$$



$$\int_{-\infty}^{\infty} u(t, x) dx = 1 \quad (\text{大に於て})$$

あてでやる

例

$$\theta(x, y) = 4 \operatorname{Tan}^{-1} \exp\left(ax + \frac{y}{a}\right) \quad (a \text{ は定数})$$

($\neq 0$)

②

$$\theta_{xy} = \sin \theta$$

Sine Gordon
equation

課題

- ▶ 講義資料や講義の誤りの指摘
- ▶ 講義内容に関する質問

提出：所定の用紙で T2SCHOLA に
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