微分積分学第一 (LAS.M101-06)

重積分の考え方

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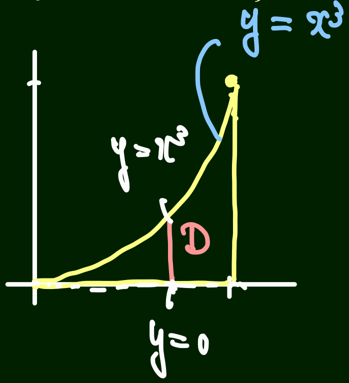
$$S = \iint_D \frac{x^2 y}{1+x^6}\,dx\,dy \qquad D = \{(x,y)\,;\,0 \leqq y \leqq x^3, 0 \leqq x \leqq 1\}$$

$$S = \int_0^1 dx \int_0^{x^3} \frac{x^2 y}{1+x^6}\,dy$$

$$= \int_0^1 dx\,\frac{x^2}{1+x^6} \cdot \left[\frac{1}{2} y^2\right]_0^{x^3}$$

$$= \int_0^1 \frac{1}{2}\,\frac{x^8}{1+x^6}\,dx$$



$$S = \int_0^1 \frac{1}{2} \frac{x^8}{1+x^6}\,dx$$

$$= \frac{1}{6}\int_0^1 \frac{u^2}{1+u^2}\,du$$

$$= \frac{1}{6}\int_0^1 \left(1 - \frac{1}{1+u^2}\right)du$$

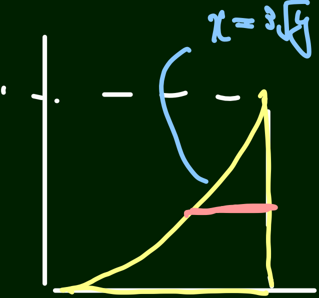
$$= \frac{1}{6}\left(1 - \left[\tan^{-1} u\right]_0^1\right)$$

$$= \frac{1}{6}\left(1 - \frac{\pi}{4}\right)$$

$$u = x^3$$

$$du = 3x^2\,dx$$

$$S = \int_0^1 dy \int_{\sqrt[3]{y}}^1 \frac{x^2 y}{1+x^6}\,dx$$



$$= \int_0^1 y\,dy \int_{\sqrt[3]{y}}^1 \frac{x^2}{1+x^6}\,dx$$

$$= \int_0^1 y\,dy \left(\frac{1}{3} \int_y^1 \frac{1}{1+u^2}\,du \right) \qquad u = x^3$$

$$= \frac{1}{3}\int_0^1 \underbrace{y\left(\mathrm{Tan}^{-1} 1 - \mathrm{Tan}^{-1} y\right)}_{\text{部分積分}} dy$$

例題

$$V=\iiint_D 1\,dx\,dy\,dz,\qquad D=\{(x,y,z)\,;\,x^2+y^2+z^4\leqq 1\,;\,x,y,z\geqq 0\}\subset\mathbb{R}^3$$

・yとzを固定すると

$$0\leqq x\leqq\sqrt{1-y^2-z^4}$$

これが空でないためには

$$1-y^2-z^4\geqq 0$$

・zを固定すると

$$0\leqq y\leqq\sqrt{1-z^4}$$

・これが空でない条件　$0\leqq z\leqq 1$

$$V = \int_0^1 dz \int_0^{\sqrt{1-z^4}} dy \int_0^{\sqrt{1-y^2-z^4}} dx$$

$$= \int_0^1 dz \underline{\int_0^{\sqrt{1-z^4}} dy \cdot \sqrt{1-y^2-z^4}} \qquad 1-z^4-y^2$$

$$= \int_0^1 dz \int_0^{\sqrt{1-z^4}} \sqrt{1-z^4}\sqrt{1-\left(\frac{y}{\sqrt{1-z^4}}\right)^2}\, dy$$

$$= \int_0^1 dz \sqrt{1-z^4} \int_0^1 \sqrt{1-u^2} \cdot \sqrt{1-z^4}\, du \qquad \frac{y}{\sqrt{1-z^4}} = u$$

$$= \int_0^1 dz\,(1-z^4) \underbrace{\int_0^1 \sqrt{1-u^2}\,du}_{\frac{\pi}{4}}$$

$$= \frac{\pi}{4} \int_0^1 (1-z^4)\,dz$$

$$= \frac{\pi}{4}\left(1-\frac{1}{5}\right) = \frac{\pi}{5} \quad \square$$

課題

- 講義資料や講義の誤りの指摘
- 講義内容に関する質問

提出：所定の用紙で T2SCHOLA に

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