微分積分学第一 (LAS.M101-06)

広義積分

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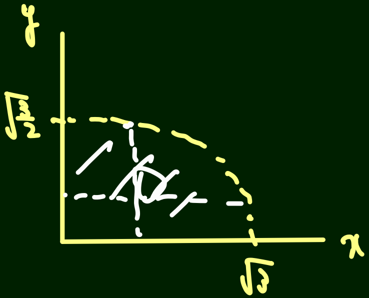
中間試験のコメント：問題 B

$$\frac{x^2}{3} + \frac{y^2}{\frac{3}{2}} \leqq 1$$

$$I := \iint_D \frac{x}{1 + x^2 + 2y^2}\, dx\, dy, \qquad D = \{(x, y) \mid x^2 + 2y^2 \leqq 3,\, x, y \geqq 0\}$$

$$\int_0^{\sqrt{\frac{3}{2}}} dy \int_0^{\sqrt{3-2y^2}} dx$$

$$\int_0^{\sqrt{3}} dx \int_0^{\sqrt{\frac{3-x^2}{2}}} dy$$



$$I := \iint_D \frac{x}{1+x^2+2y^2}\,dx\,dy, \qquad D = \{(x,y) \mid x^2+2y^2 \leqq 3,\ x,y \geqq 0\}$$

$$x = \sqrt{2}u\cos v, \qquad y = u\sin v$$

$$= \iint_{\tilde{D}} \frac{\sqrt{2}u\cos v}{1+2u^2} \boxed{}\, du\,dv$$

$$\left|\frac{\partial(x,y)}{\partial(u,v)}\right|$$

Jacobi行列式

$$= \left|\det\begin{pmatrix} x_u & x_v \\ y_u & y_v \end{pmatrix}\right| = |\sqrt{2}u|$$

$$\tilde{D} = \left\{(u,v) \,\middle|\, 0 \leqq u \leqq \sqrt{\tfrac{3}{2}},\ 0 \leqq v \leqq \tfrac{\pi}{2}\right\}$$

$-\sqrt{2}$：式が 1:1 に合わない

$$I = \iint_{\tilde{D}} \frac{\sqrt{2}\, u \cos v}{1+2u^2} \sqrt{2}\, |u| \, du\, dv$$

$\overset{u}{\shortparallel}$

$$\tilde{D} = \left[0, \sqrt{\tfrac{3}{2}}\right] \times \left[0, \tfrac{\pi}{2}\right]$$

$$= \int_0^{\frac{\pi}{2}} dv \int_0^{\sqrt{\frac{3}{2}}} \frac{2u^2 \cos v}{1+2u^2}\, du$$

$$= \int_0^{\frac{\pi}{2}} \cos v \, dv \int_0^{\sqrt{\frac{3}{2}}} \left(1 - \frac{1}{1+2u^2}\right) du$$

$=$

$\cancel{\dfrac{dx}{du}}$

$$F(u,v) = \bigl(x(u,v), y(u,v)\bigr) = (e^u \operatorname{sech} v, e^u \tanh v)$$

$$\begin{pmatrix} \dfrac{\partial x}{\partial u} = e^u \operatorname{sech} v & \dfrac{\partial x}{\partial v} = e^u \operatorname{sech} v \tanh v \\ \dfrac{\partial y}{\partial u} = e^u \tanh v & \dfrac{\partial y}{\partial v} = e^u \operatorname{sech}^2 v \end{pmatrix}$$

$$\begin{pmatrix} \dfrac{\partial \tilde f}{\partial u} & \dfrac{\partial \tilde f}{\partial v} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial f}{\partial x} & \dfrac{\partial f}{\partial y} \end{pmatrix} \begin{pmatrix} x_u & x_v \\ y_u & y_v \end{pmatrix}$$

$$\tilde f(u,v) = f\bigl(x(u,v), y(u,v)\bigr)$$

中間試験のコメント：問題 A

$$F(u,v) = \big(x(u,v), y(u,v)\big) = (e^u \operatorname{sech} v, e^u \tanh v)$$

$$\frac{1}{\frac{\partial x}{\partial u}} \times \qquad \begin{pmatrix} u_x & u_y \\ v_x & v_y \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial \tilde{f}}{\partial u} & \frac{\partial \tilde{f}}{\partial v} \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{pmatrix} \begin{pmatrix} x_u & x_v \\ y_u & y_v \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{pmatrix} = \begin{pmatrix} \frac{\partial \tilde{f}}{\partial u} & \frac{\partial \tilde{f}}{\partial v} \end{pmatrix} \underbrace{\begin{pmatrix} x_u & x_v \\ y_u & y_v \end{pmatrix}^{-1}}$$

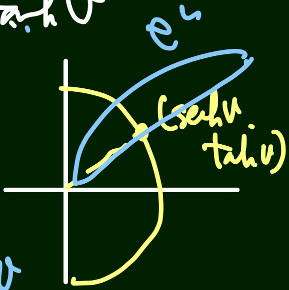
- $$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$ ✓

$\Delta f = 0 \quad \cdots \quad f = a\theta + b \qquad (x = r\cos\theta,\ y = r\sin\theta)$

$f = (\theta$のみ$)$

- $x = e^u \operatorname{sech} v \qquad y = e^u \tanh v$

$$\operatorname{sech}^2 v + \tanh^2 v = 1$$



- $\Delta F(v(x,y)) = 0$

$$\theta = \operatorname{Tan}^{-1}\frac{y}{x} = \operatorname{Tan}^{-1}\sinh v$$