Advanced Topics in Geometry A1 (MTH.B405)

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Notice

- Sorry that the uploaded files of the blackboards on April 25 was incorrect. I've uploaded the correct versions.

- Deadline of today's homeworks: May 13 (Tue) 10:00JST.

Q and A

Q: I got the solution $x(t) = -\frac{\cos t}{\sin t}$ ($t \in (0, \pi)$). The width of this domain π is the same as the initial value $x(0) = 0$. Isn't the width of the maximal solution for the same differential equation the same for different initial values? (The expectation may change only for special values.)

A: No.
Counterexample:

$$\frac{dx}{dt} = t(1 + x^2), \qquad x(0) = a$$

Q and A

Q: In Exercise 2-1, for example, the equation is solved without giving the domain of x. Is there a situation that one wants to know "a function whose domain is not known (not determined) but whose differential equation is known"? Examples in physics are easy to understand.

A: Usually, the domain of definition of a solution of a differential equation is determined a posteriori.

e.g. the equation of geodesics in surface theory and/or Riemannian geometry.