14 submissions of homeworks.

Advanced Topics in Geometry A1 (MTH.B405)

Kotaro Yamada
kotaro@math.sci.isct.ac.jp
http://www.official.kotaroy.com/class/2025/geom-a1

Institute of Science Tokyo

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Q and A

Q: I have some doubts about Theorem (Poincaré's lemma). What is the meaning of $d\omega = 0$? And also, what is the meaning of "unique up to additive constants". Does this mean that the form of such function is: $f + c(\text{constant})$ which satisfies $df = \omega$?

- "2-dim" $\omega = a\,dx + b\,dy$

$\Rightarrow d\omega = (b_x - a_y)\,dx \wedge dy$

$d\omega = 0 \Leftrightarrow b_x = a_y$ by definition

Q and A

Q: Though I understand the proof of Poincaré's lemma, there is a formulation of sentence that is strange to me. When in the last paragraph it is written that "Proposition 2.8 yields $\xi (= \det \xi)$ never vanishes", the important point that we are keeping from Proposition 2.8 in this case is not that $\xi = \det \xi$ right? Since it is a well-known fact, the important thing that Proposition 2.8 allows us to exploit even in the case of "1×1 matrices" is that $\xi = (\det \xi =) \xi(0) \exp \int_{t_0}^{t} \alpha(\tau)\, d\tau$ and thus is always of same sign as $\xi(0)$. This sentence makes me wonder if my understanding is correct?