May 23, 2025
Kotaro Yamada
kotaro@math.sci.isct.ac.jp

Info. Sheet 5; Advanced Topics in Geometry A1 (MTH.B405)

Corrections

- The URL of Zoom Recording was wrong. The correct URL has been sent.
- Lecture Note, page 19, one line before eq. (4.10): *vector product of* $\boldsymbol{x} \times y$ $\Rightarrow$ *vector product* $\boldsymbol{x} \times y$
- Lecture Note, page 20, line 9: Cauchy-Schwartz $\Rightarrow$ Cauchy-Schwarz
- Lecture Note, page 21, line 5: it hold that $\Rightarrow$ it holds that
- Lecture Note, page 21, eq. (4.15):
  $A = (A_i^j),\ A_i^j = \sum_{k=1}^{2} g^{jk} h_{ki} \Rightarrow A = (A_j^i),\ A_j^i = \sum_{k=1}^{2} g^{ik} h_{kj}$ (Usage of the indices i and j is unified that "i" for the number of row and "j" for the column.)
- Lecture Note, page 22, line 10: both side $\Rightarrow$ both sides
- Lecture Note, page 23, line 16: using orthonormal frames. $\Rightarrow$ using orthonormal frames is given.
- Lecture Note, page 23, line 20: for a simplicity $\Rightarrow$ for simplicity
- Lecture Note, page 26, Problem 4-1: u_1, u_2 $\Rightarrow$ u^1, u^2
- Page 8 of Handout/Blackboard B on 20250516: $-d\nu \cdot dp == L \dots \Rightarrow -d\nu \cdot dp = L \dots$ $\Rightarrow$
- Page 8 of Handout/Blackboard C on 20250516: Ex. 3-2 $\Rightarrow$ Ex. 4-2

Students' comments

- 4-1 はもっとスッキリしたやりかたがあったりするのかなあ
  I wonder if there is a simpler way to do 4-1.

  Lecturer's comment What do you think?

- I think the course is good. No request this time.

  Lecturer's comment I see.

Q and A

Q 1: My question is actually about the exercise 4-2. Is it normal that there is a possibility for det $\widehat{I}$ vanish? Wouldn't that be contradictory to the definition of p as a parametrization according to the Cauchy-Schuwarz inequality as exploited in (4.8).

A: Yes, it causes a problem for the regularity of the parametrization. The point where det $\widehat{I}$ vanishes is called **singularities** or **singular points** of the surface, at which the usual surface theory can not be applied. I'll explain it briefly today, and in the lectures of 2Q. In the case of Ex. 4-2, the assumption like as $\theta \in (0, \pi)$ should be required.

Q 2: Is there a geometric intuition behind the weingarten matrix or is it just an abstract definition?

A: The Weingarten matrix is defined to satisfy $(\nu_u, \nu_v) = -(p_u, p_v)A$. Namely, the "variation" of the unit normal vector field is expressed in terms of A.lo

Q 3: 主曲率は一番曲がっている方の曲がり具合，ガウス曲率はその積だから，符号から大体の形がわかると思っています．一方，平均曲率は図形的に何に対応するかイマイチわかりません．何なんでしょうか．そもそもこのような直感的理解が可能な代物なのでしょうか．
Since the principal curvatures are the curvature of the most curved direction and the Gaussian curvature is the product of the two of them, I think I can get roughly a shape of surface from the sign of Gaussian curvature. On the other hand, I am not sure what the mean curvature geometrically. Is it something that can be intuitively understood?

A: Several meanings of the mean curvature will be introduced the 7th lecture, through investigation of surfaces of constant mean curvature.

Q 4: What is the difference between Frenet frame and Gauss frame? Do we have any application example of Gauss frame?

A: The Frenet frame is for space curves, and the Gauss frame is for surfaces. An application of the Gauss frame will be introduced in the 7th lecture, and lectures on 2Q.

Q 5: $\check{I\!I}$ で $\bigl((\boldsymbol{e}_3)_u, (\boldsymbol{e}_3)_v\bigr) = -(\boldsymbol{e}_1, \boldsymbol{e}_2)\check{I\!I}$ と $-$ がつくようにとったのはなぜですか？
In the definition of $\check{I\!I}$, why did you take the minus sign?

A: It is a traditional custom, but some people do not take the minus sign.

Q 6: Gauss-Weingarten の公式を Gauss frame と Adapted Frame の両方で見ましたが, Adapted Frame はどのような場面で使いますか？
We have seen the Gauss-Weingarten formula in both Gauss frame and Adapted Frame. In what situations do you use Adapted Frame?

A: It will be used in the lecture of 2Q.

Q 7: The Gauss frame 以外の frame は今回紹介された (lecture note にのっている) もの以外にも複数あるのでしょうか.
Are there more frames different from the Gauss frame, and ones given in the lecture note?

A: Regardless of these, we take the appropriate frame for each problem.

Q 8: Handouts C の "index formulation" にかかれている $(g^{ij}) = (g_{ij})^{-1}$ とう式は g^{ij} が g_{ij} の逆元 (山田注：逆数のこと？) という意味ではなく, (i,j) 成分が g_{ij} の行列の逆行列の (i,j) 成分が g^{ij} という認識であっていますか.
Does the notion $(g^{ij}) = (g_{ij})^{-1}$ mean that g^{ij} is the (i,j)-component of the inverse matrix of the matrix (g_{ij})?

A: Yes. (g_{ij}) denotes the matrix whose components are g_{ij}.