Advanced Topics in Geometry A1 (MTH.B405)

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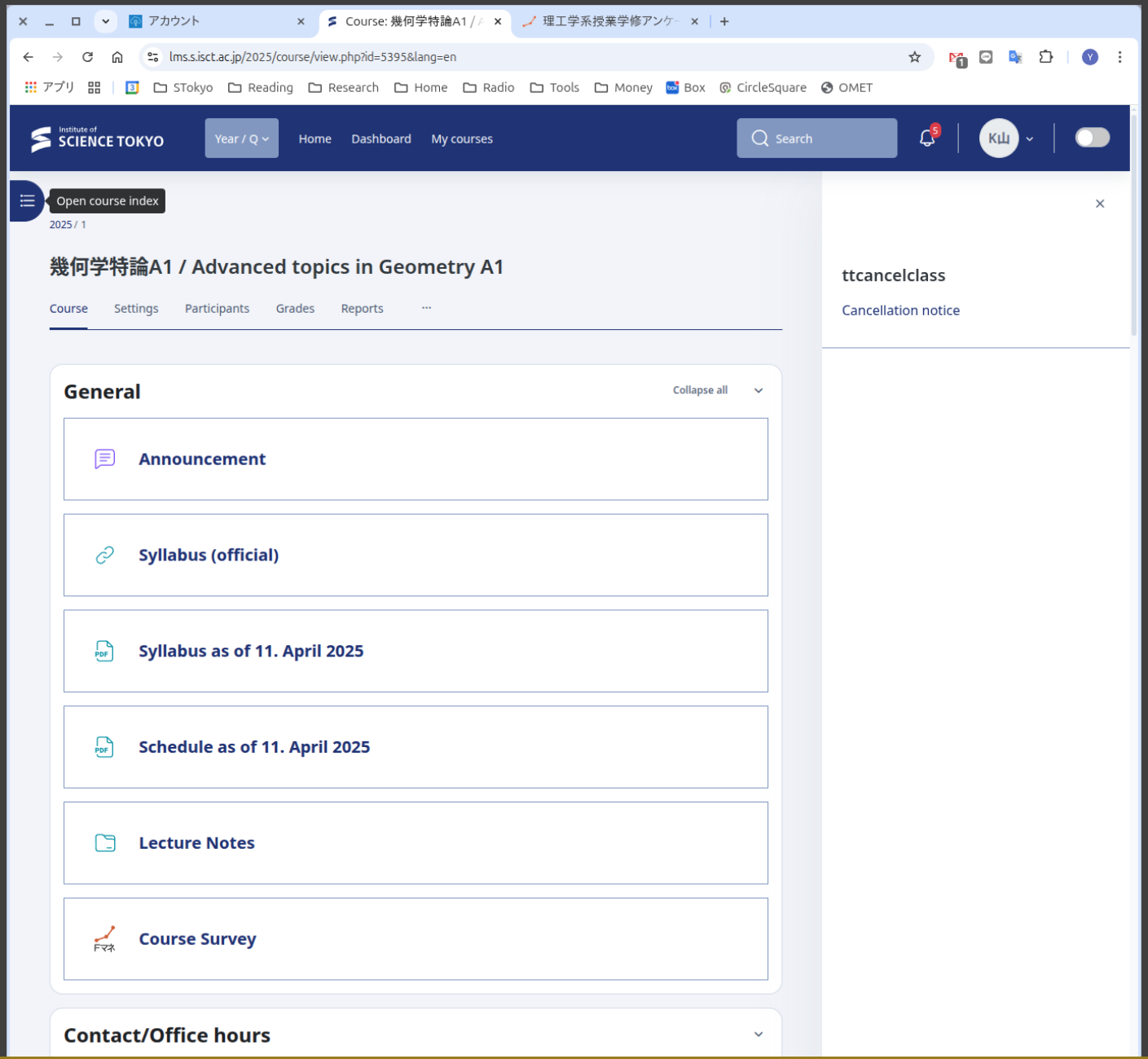
2025/06/06

Notice

- Today is the final class of MTH.B405. Thank you for attending and cooperating the course.
- Please fill the form “Course Survey” in LMS.

Course Survay



Q and A

Q: Do we have any example when (g_{ij}, h_{ij}) does not satisfy Gauss equation or Codazzi equation, and show that such surface does NOT exist?

Q: What happens if two symmetric matrices $\widehat{I}$ and $\widehat{II}$ with components that are real-valued C^∞-functions on U satisfy only Gauss equation? Or only Codazzi's? Can anything be said about the existence of regular surfaces p with fitting fundamental forms in such cases?

$\nexists$ surface ☹ Gauss & Codazzi $\Leftrightarrow$ Integrability of Gauss-Weingarten

p : surface $\quad$ ν : its unit normal

Gauss - Weingarten :

$$\mathcal{F}_u = \mathcal{F}\Omega, \quad \mathcal{F}_v = \mathcal{F}\Lambda, \quad \mathcal{F} = (p_u, p_v, \nu)$$

Integrability (or compatibility)

$\Leftrightarrow \quad \mathcal{F}_{uv} = \mathcal{F}_{vu}$

$\Leftrightarrow \quad p_{uv} = p_{vu}, \quad \nu_{uv} = \nu_{vu}$

$\Leftrightarrow$ Gauss & Codazzi

Gauss or Codazzi fails $\Rightarrow$ compatibility fails

$\Rightarrow \nexists$ surface