Advanced Topics in Geometry B1 (MTH.B406)

Kotaro Yamada
kotaro@math.sci.isct.ac.jp
http://www.official.kotaroy.com/class/2025/geom-b1

Institute of Science Tokyo

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<u>Correction</u>

Problem 2-1: $z'' = z \to x'' = x$

Student's comments

- The exercises were harder than they looked. I couldn't solve 1-2, so I look forward to the explanation.

(Sorry. I forgot add Hints.)

Q and A

~~A~~ Q: In the lecture, the distance formula was derived from the fact it is invariant under congruence, but was distance obtained historically in this way? If so, how did you come up with congruence?

may be true (in some sence)

- lines
- "the property to be a line is invariant under congruence"

(cf. Klein's Erlangen program)

Q and A

BOOK I. PROPOSITIONS.

PROPOSITION 1.

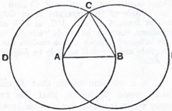
On a given finite straight line to construct an equilateral triangle.

Let AB be the given finite straight line.

Thus it is required to construct an equilateral triangle on the straight line AB.

With centre A and distance AB let the circle BCD be described; [Post. 3]

again, with centre B and distance BA let the circle ACE be described; [Post. 3]



and from the point C, in which the circles cut one another, to the points A, B let the straight lines CA, CB be joined. [Post. 1]

Now, since the point A is the centre of the circle CDB,

AC is equal to AB. [Def. 15]

Again, since the point B is the centre of the circle CAE,

BC is equal to BA. [Def. 15]

But CA was also proved equal to AB;

therefore each of the straight lines CA, CB is equal to AB.

And things which are equal to the same thing are also equal to one another; [*C. N.* 1]

therefore CA is also equal to CB.

Therefore the three straight lines CA, AB, BC are equal to one another.

Therefore the triangle ABC is equilateral; and it has been constructed on the given finite straight line AB.

(Being) what it was required to do.

congruence

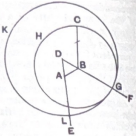
[Euc11]

Q and A

PROPOSITION 2.

To place at a given point (as an extremity) a straight line equal to a given straight line.

Let A be the given point, and BC the given straight line. Thus it is required to place at the point A (as an extremity) a straight line equal to the given straight line BC.



From the point A to the point B let the straight line AB be joined; [Post. 1]

and on it let the equilateral triangle DAB be constructed. [I. 1]

Let the straight lines AE, BF be produced in a straight line with DA, DB; [Post. 2]

with centre B and distance BC let the circle CGH be described; [Post. 3]

and again, with centre D and distance DG let the circle GKL be described. [Post. 3]

Then, since the point B is the centre of the circle CGH,

BC is equal to BG.

Again, since the point D is the centre of the circle GKL,

DL is equal to DG.

And in these DA is equal to DB;

therefore the remainder AL is equal to the remainder BG. [C.N. 3]

But BC was also proved equal to BG;

therefore each of the straight lines AL, BC is equal to BG.

And things which are equal to the same thing are also equal to one another; [C.N. 1]

therefore AL is also equal to BC.

Therefore at the given point A the straight line AL is placed equal to the given straight line BC.

(Being) what it was required to do.

[Euc11]

Q and A

Q: The book I referred used the projective model, but how does it relate to the upper half-plane as well?

- $\exists$ many models of non-Euclidean geom
- Lectures 6 & 7