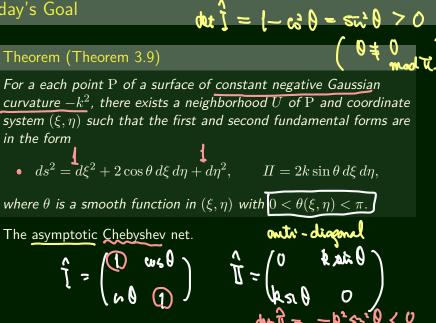
(General construction of pseudosphenical enfance.)

Advanced Topics in Geometry B1 (MTH.B406) Asymptotic Chebyshev nets

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· Choose pourameteurs of surfaces.



Asymptotic Coordinate system

p(u,v): a regular parametrization of a surface in \mathbb{R}^3 :

▶ (u, v) is an asymptotic coordinate system \Leftrightarrow $\Pi = 2M \, du \, dv$

Proposition (Asymptotic Coordinate system; Prop. 3.8)

Let $p: U \to \mathbb{R}^3$ be a regular parametrization of a surface in \mathbb{R}^3 whose Gaussian curvature is <u>negative</u> on U. Then for each $P \in U$, there exists an asymptotic coordinate system on a neighborhood of P.

$$I = L du^{2} + 2M du dv + N dv^{2} \quad K < 0$$

$$\Rightarrow \underline{W} - \underline{W}^{2} < 0$$
We may assume (without loss of generality)
$$= \frac{W^{2}}{W} = 0$$

$$\Rightarrow \underline{W} = 0 \Rightarrow (\underline{W}, \underline{V}) \mapsto (\underline{V}, \underline{W}) \Rightarrow \underline{L} \neq 0$$

$$: \underline{U} = N = 0; \quad \underline{M} \neq 0$$

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$$: \underline{U} = S + t \qquad \underline{I} = 2M (\underline{d}S - \underline{d}L^{2}) at P$$

$$= L \left((\underline{d}u^{2} + \frac{2M}{L} du dv + \frac{M^{2}}{L^{2}} dv^{2} - \frac{M^{2} - \underline{M}}{L^{2}} dv^{2} \right)$$

$$= L \left((\underline{d}u + \frac{M}{L} dv)^{2} - \frac{M^{2} - \underline{M}}{L^{2}} dv^{2} \right)$$

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$$= I = L \left((d du + g dv) (V du + \delta' dv) \right)$$

$$d\xi = \left(\varphi \left(\frac{\lambda du + g dv}{\lambda du} \right) \right)$$

$$d\eta = \psi \left(Y du + F dv \right)$$

$$\int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} dv$$

$$\int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} dv dv$$

Integrating factor

Lemma (A special case of Caratheodory's)

Let $\omega = \alpha \, du + \beta \, dv$ be a 1-form defined on a domain U of the uv-plane \mathbb{R}^2 , where \mathbb{R} and \mathbb{Q} are functions in (u, v). Assume $(\alpha, \beta) \neq (0, 0)$ at $P \in U$. Then there exists a neighborhood $V \subset U$ of P and functions φ and ξ on V such that

$$\varphi \omega = d\xi, \qquad \varphi(\mathbf{Q}) \neq 0 \quad \text{for} \quad \mathbf{Q} \in V.$$

~ aq 25 qu = dξ ⇔ wrdu = 0"

Proof of essectance of Asymp. Chappeder nut ds² = E du² + 2F du dv + 6 dv² I = 2M durde ⇒ Apply Problem 5-1 in 1Q Ev= Gu = 0 < Codazzi eg $F = (e(u))^2 \quad f = (q(u))^2$ $E du^{2} = (e du)^{2} = du^{2}$ G duz= dyy= ∫gdvr I= ledu

Example $\emptyset = \emptyset(\underline{\mathfrak{z}}, \eta)$

$$p(u, v) = (\operatorname{sech} v \operatorname{cos} u, \operatorname{sech} v \operatorname{sin} u, v - \tanh v)$$

$$ds^{2} = \operatorname{sech}^{2} v du^{2} + \tanh^{2} v dv^{2},$$

$$II = -\operatorname{sech} v \tanh v (du^{2} - dv^{2}).$$

$$= -\operatorname{sech} v \tanh v (du - dv) (du + dv')$$

$$d\xi = du - dv \qquad \xi = u - v$$

$$d\eta = du + dv \qquad \eta = u + v$$

$$u = \frac{1}{2} (\xi + \eta)$$

$$v = \frac{1}{2} (-\xi + \eta)$$

Exercise 3-1

Problem

Let a and b be real numbers with $a \neq 0$ and

 $p(u, v) = a(\operatorname{sech} v \cos u, \operatorname{sech} v \sin u, v - \tanh v) + b(0, 0, u).$

Find a coordinate change $(u, v) \mapsto (\xi, \eta)$ to an asymptotic Chebyshev net for p, and give an explicit expression of θ as a function in (ξ, η) .

Exercise 3-2 · (wywwww.)

Let $(\underline{\xi}, \eta)$ be an asymptotic Chebyshev net on a surface. Assume another parameter (x, y) is also an asymptotic Chebyshev net. Prove that (x, y) satisfies

$$(x,y) = (\pm \xi + x_0, \pm \eta + y_0)$$
 or $(x,y) = (\pm \eta + x_0, \pm \xi + y_0)$

where x_0 and y_0 are constants.